This version of the article has been accepted for publication in *Erkenntnis*, after peer review but is not the Version of Record and does not reflect post-acceptance improvements, or any corrections. Please cite the published version forthcoming.



# Transitivity When the Same are Distinct

August 28, 2021

**Abstract**

It is widely assumed that the identity relation is, among other things, transitive. Some have proposed that the identity relation might hold between objects contingently or occasionally. If, on those proposals, identity is shown to not be transitive, then there is reason to reject such proposals. One such argument attempts to show that the identity relation on such proposals violates transitivity in cases of 'simultaneous' fissions and fusion. I argue that, even in those cases, contingent identity and occasional identity are transitive.

## 1 Introduction

In this article I consider objections raised by Ralph Bader (2012) against the Contingent Identity and Occasional Identity views held by André Gallois (1998).[1] Bader argues that these views violate a widely held principle regarding identity, specifically the principle that says the identity relation is transitive. I argue that one can reformulate Transitivity of Identity to reply to Bader's objections.

In subsection 1.1, I define views and principles. In subsection 1.2, I introduce the case that features throughout. In section 2, I review Bader's arguments that

Contingent Identity and Occasional Identity are about relations that are not transitive. In section 3, I reformulate a transitivity principle for a different relation and use this reformulation to show how Contingent Identity and Occasional Identity theorists ought to formulate principles of transitivity in general. Finally, in section 4, I show how Contingent Identity and Occasional Identity theorists can reply to Bader's objections.

### 1.1 Definitions

Frege (1892, p. 26) famously said of identity that it is "a relation ...of a thing to itself, and indeed one in which each thing stands to itself but to no other thing." One way to further explicate the relation is to identify principles that it obeys. Call such principles `Principles of Identity`.[2] One set of these principles, which I will call `Logical Principles of Identity`, come from the assumption that identity is an equivalence relation (and arguably, *the* paradigmatic equivalence relation). As such, identity is thought to be reflexive, symmetric, and transitive.[3]

Presently, of particular interest is transitivity. A relation is said to be transitive when, for three (not necessarily distinct) objects, if the first stands in the relation to the second and the second stands in the relation to the third, then the first stands in the relation to the third. More precisely and with respect to identity:

`Transitivity of Identity` $\forall x \forall y \forall z([(x = y) \wedge (y = z)] \rightarrow x = z)$

Another group of Principles of Identity are what I will call the `Metaphysical Principles of Identity`. Among these are the principles that say that the identity relation is absolute, necessary, eternal, determinate, and one-to-one. Presently, of particular interest are the principles that say identity is necessary and eternal.[4]

A relation holds necessarily just in case, necessarily, if it holds between objects it could not have failed to hold between them. More precisely and with respect to identity:

`Necessity of Identity:` $\Box\forall x\forall y[x = y \to \Box(x = y)]$

A relation holds eternally just in case, necessarily, if it holds between objects at one time, it holds between them at all times (at least, when the objects exist). More precisely and with respect to identity, where $t$ ranges over times:

`Eternality of Identity:` $\Box\forall x\forall y\forall t[\exists t_1(t_1 : x = y) \to t : x = y]$

If one holds what I call the `Standard View of Identity`, then one is committed to the truth of the Logical Principles of Identity, Metaphysical Principles of Identity, and Leibniz's Law.[5] See, for example, Hawthorne 2003 for a defense of the Standard View of Identity. Let a view be a `Non-Standard View of Identity` if it rejects one of the Principles of Identity.

Two such views are `Contingent Identity` and `Occasional Identity`. According to Contingent Identity, the Necessity of Identity is false. According to the view, identity can hold between objects at some worlds and not at others. That is, it is possible that there are some objects that are identical, but might have been distinct. According to Occasional Identity, the Eternality of Identity is false. According to the view, identity can hold between objects at some times and not at others. That is, it

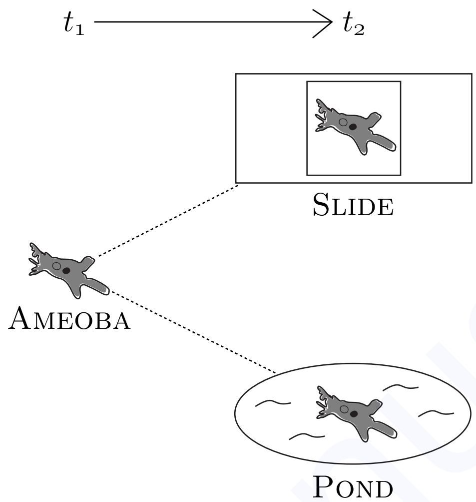


Figure 1: Amoeba division

is possible that there are some objects that are identical at one time, but distinct at another. Put more precisely:

`Contingent Identity:` $\Diamond\exists x\exists y[x = y \wedge \Diamond\, x \neq y]$

`Occasional Identity:` $\Diamond\exists x\exists y\exists t\exists t'[\text{ at } t\text{: } x = y \wedge \text{ at } t'\text{: } x \neq y]$

## 1.2 The case of Amoebic Division

What follows concerns the modal and temporal versions of the following case from Gallois (1998, §1.6). Imagine an amoeba called AMOEBA undergoes a division such that there are two amoebas at a later time. One of them, called SLIDE, ends up under a microscope. The other, called POND, ends up in a pond. SLIDE and POND seem to be distinct objects. See Figure 1 for a representation of the division.

Gallois maintains that before the division, say at $t_1$, SLIDE and POND are identical in virtue of being AMOEBA. But after the division, say at $t_2$, SLIDE and POND are distinct. As such this is a purported case of Occasional Identity.[6] The objection considered in the next section expands on the case to argue that if Occasional Identity is true, then identity is not transitive. The general strategy of this objection is to argue against a Non-Standard View of Identity which rejects one of the Metaphysical Principles of Identity (in this case Eternality of Identity), by arguing that doing so involves rejecting one of the Logical Principles of Identity (in this case Transitivity of Identity). I take it that an assumption underlying this form of objection is that it is more costly to reject one of the Logical Principles of Identity than one of the Metaphysical Principles of Identity.

## 2 Objections from Transitivity

In subsection 2.1 and subsection 2.2, I review common and structurally similar arguments against Occasional Identity and Contingent Identity, respectively. In subsection 2.3, I review how reformulations of transitivity are used to respond to these arguments. In subsection 2.4, I review how, granting these reformulations, Bader objects to Occasional Identity and Contingent Identity.

## 2.1 The Argument from Transitivity against Occasional Identity

In his defense of Occasional Identity, Gallois (1998) considers the following objection to the view. Assume that the amoebic division described above (subsection 1.2) is in fact a case of Occasional Identity. Here is the objection:

1.1 at $t_1$: Slide = Pond $\wedge$ at $t_2$: Slide $\neq$ Pond (assumption for reductio).

1.2 That which is Pond at $t_1$ = that which is Slide at $t_1$ (from 1.1).

1.3 That which is Slide at $t_1$ = that which is Slide at $t_2$ (by Reflexivity of Identity).[7]

1.4 That which is Pond at $t_2$ = that which is Pond at $t_1$ (by Reflexivity of Identity).

1.5 That which is Pond at $t_1$ = that which is Slide at $t_2$ (by Transitivity of Identity, 1.2, and 1.3).

1.6 That which is Pond at $t_2$ = that which is Slide at $t_2$ (by Transitivity of Identity, 1.4, and 1.5).

1.7 at $t_2$: Pond = Slide (From 1.6, which contradicts the assumption in 1.1.)[8]

## 2.2 The Argument from Transitivity against Contingent Identity

In defending Contingent Identity, Gallois does not consider an argument from transitivity against the view. However, it is not difficult to construct an objection to Contingent Identity that is analogous to the one just given against Occasional Identity. In fact, Bader (2012, p. 145) presents an argument like the one that follows. It comes from the statue case from Allan Gibbard (1975). In it, the statue, GOLIATH, and the lump it is made of, LUMP$_1$, are presumed to be identical in virtue of having the same persistence conditions (they are brought into existence at the same time and, later, out of existence at the same time). But, argues Gibbard, we are to imagine that the clay of which GOLIATH is made might be compressed into a non-statue form (Gibbard, 1975, p. 191). As a purported case of Contingent Identity, GOLIATH and LUMP$_1$ are identical, but might not have been if the compression were to happen.

2.1 at @[9]: GOLIATH = LUMP$_1$ $\wedge$ at $w_1$: GOLIATH $\neq$ LUMP$_1$ (assumption for reductio).

2.2 That which is GOLIATH at @ = that which is LUMP$_1$ at @ (from 2.1).

2.3 That which is LUMP$_1$ at @ = that which is LUMP$_1$ at $w_1$ (by Reflexivity of Identity).[10]

2.4 That which is GOLIATH at $w_1$ = that which is GOLIATH at @ (from Reflexivity of Identity).

2.5 That which is GOLIATH at @ = that which is $\text{LUMP}_1$ at $w_1$ (by Transitivity of Identity, 2.2, and 2.3).

2.6 That which is GOLIATH at $w_1$ = that which is $\text{LUMP}_1$ at $w_1$ (by Transitivity of Identity, 2.4, and 2.5).

2.7 at $w_1$: GOLIATH = $\text{LUMP}_1$ (From 2.6, which contradicts the assumption in 2.1).

## 2.3 Replying to Transitivity Arguments

Gallois replies to the argument against Occasional Identity by objecting to the applications of transitivity in 1.5 and 1.6. The reason is that the formulation of transitivity presupposes the Standard View of Identity. The Standard View of Identity assumes that, if the relation holds at one time, then it must hold at all times. This is precisely what Occasional Identity theorists reject (Gallois, 1998, pp. 76–9). However, Occasional Identity theorists are committed only to a formulation of Transitivity of Identity that relativizes transitivity to particular times.[11] Here is the formulation Gallois claims that those who endorse Occasional Identity ought to hold:

$\texttt{Transitivity of Identity}_{\texttt{t}}$: $\forall x \forall y \forall z \forall t[(\text{ at } t : x = y \wedge \text{at } t\text{: } y = z) \rightarrow \text{at } t : x = z]$

That is, for all objects and times, if (i) it is the case that one object and a second object are identical at one time and (ii) it is the case that the second object and a third object are identical at that same time, then it is the case that the first object and the third object are identical at that same time. Adopting Transitivity of Identity$_\mathrm{t}$ as the reformulation of Transitivity of Identity allows those who endorse Occasional Identity to object to the argument in subsection 2.1. If transitivity is fully captured by Transitivity of Identity$_\mathrm{t}$, one cannot rely on transitivity to make inferences about identities at distinct times. Proponents of Occasional Identity can reply by pointing out that the identities in 1.2, 1.3, and 1.4 are identities across distinct times. These identities are not the antecedents of instances of Transitivity of Identity$_\mathrm{t}$. So the argument fails on this formulation of transitivity, because the moves to 1.5 and 1.6 are not correct applications of *modus ponens*.

Similarly, Bader points out that proponents of Contingent Identity can appeal to the following reformulation of Transitivity of Identity that relativizes with respect to worlds to block the objection to Contingent Identity at lines 2.5 and 2.6.

`Transitivity of Identity`$_\mathtt{w}$: $\forall x \forall y \forall z \forall w[(\text{ at } w : x = y \wedge \text{ at } w\text{: } y = z) \rightarrow \text{ at } w : x = z]$

Adopting Transitivity of Identity$_\mathrm{w}$ as the reformulation of Transitivity of Identity allows those who endorse Contingent Identity to object to the argument in subsection 2.2. If transitivity is fully captured by Transitivity of Identity$_\mathrm{w}$, one cannot rely on transitivity to make inferences about identities at distinct worlds. Proponents of Contingent Identity can reply by pointing out that the identities in 2.2, 2.3, and 2.4 are identities across distinct worlds. These identities are not the antecedents of instances of Transitivity of Identity$_\mathrm{w}$. So the argument fails on this formulation of transitivity, because the moves to 2.5 and 2.6 are not correct applications of *modus ponens*.

## 2.4 Bader's Objections from the Transitivity of Identity

However, the above objections are not the arguments that Bader uses to object to Occasional Identity and Contingent Identity. His arguments rely on the possibility of fusions of objects occurring at the same time as fissions of those objects. Gallois (1998, Chap. 1, §VI) presents his view of Occasional Identity as a view that can explain identity puzzles. According to him, the case of AMOEBA, SLIDE, and POND is a case of Occasional Identity.

This case is what Bader would classify as a fission. Inverse cases presumably are examples of fusions, where the pre-fusion objects are distinct, but identical after the fusion. Bader assumes that Gallois takes his view to apply to cases of fusions. Gallois' example of the truncated car might be seen as such a case. Before the car loses a part there is the car and object that is the collection of car parts without this part. The Occasional Identity theorist can can argue that these objects are distinct at one time, but when the part is removed from the car, they become identical (Gallois, 1998, Chap. 1, §II).[12]

I present generalizations of Bader's arguments. He uses the possibility of teleportation and the severing of brain hemispheres to present a case of simultaneous fissions and fusion and a modally analogous case (Bader, 2012, p. 143, pp. 145–6). I do not think that Bader's reliance on this particular version of co-located fissions and fusion is compelling. It seems that, for it to be compelling, one must accept some assumptions about personal identity. However, I think that the general form of the arguments are serious objections to Occasional Identity and Contingent Identity.

### 2.4.1 Against Occasional Identity

Bader's argument against Occasional Identity begins with two objects $b$ and $d$ at at time $t_1$. There are two fissions of each of these objects between $t_1$ and $t_2$. In a simpler case, this would mean that there are now four objects at $t_2$ (a pair that at $t_1$ were identical to $b$ and another pair that at $t_1$ were identical to $d$). However, there was also one fusion that occurred when the fissions happened. One of the post-fission objects of $b$ was fused with one of the post-fission objects of $d$. This means that there are three, rather than four, objects at $t_2$. Let $a$ be the post-fission object of $b$ that did not fuse. Let $e$ be the post-fission object of $d$ that did not fuse. Let $c$ be the fusion of the post-fission objects of $b$ and $d$ (Bader, 2012, pp. 143–4). See figure Figure 2 for a representation of the case.

According to Occasional Identity, the distinct objects that result from a fission are identical prior to the fission. Conversely, objects that were distinct prior to a fusion are identical after it. So, at $t_1$, $a$ and $c$ are identical in virtue of being $b$ at $t_1$. Similarly, at $t_1$, $c$ and $e$ are identical in virtue of being $d$ at $t_1$. However, since at $t_1$, $b$ is not identical to $d$, it is not the case that, at $t_1$, $a$ is identical to $e$.

Bader's objection can then be formulated as follows.

3.1. $\forall x \forall y \forall z \forall t[(\text{ at } t : x = y \wedge \text{at } t\text{: } y = z) \rightarrow \text{at } t : x = z]$ (assumption for reductio).

3.2. at $t_1 : a = c \wedge$ at $t_1$: $c = e \wedge$ at $t_1 : a \neq e$ (from the case)

3.3. (at $t_1 : a = c \wedge$ at $t_1$: $c = e) \rightarrow$ at $t_1 : a = e$ (instance of 3.1).

3.4. at $t_1 : a = e \wedge$ at $t_1 : a \neq e$ (from 3.2 and 3.3, a contradiction).

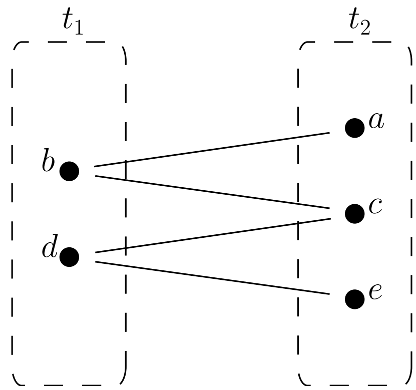


Figure 2: Fission and Fusion of Two Amoebas

3.5. $\neg\forall x\forall y\forall z\forall t[($ at $t : x = y \wedge$ at $t$: $y = z) \rightarrow$ at $t : x = z]$ (the negation of Transitivity of Identity$_\mathrm{t}$).

### 2.4.2 Against Contingent Identity

Bader objects to Contingent Identity with an analogous argument. Instead of there being a case of simultaneous fissions and a fusion, he imagines a case where two objects might have been subject to fissions and a fusion. In the modal analog, at $w_1$ objects $b$ and $d$ are distinct. In $w_2$, $b$ is distinct objects $a$ and $c$, and $d$ is distinct objects $c$ and $e$. See Figure 3 for a representation of the case.

As in the temporal case, objects $a$ and $c$ that are distinct in $w_2$ are identical in $w_1$ in virtue of being $b$ there. By the same reasoning, objects $c$ and $e$ that are distinct in $w_2$ are identical in $w_1$ in virtue of being $d$ there (Bader, 2012, pp. 144–6). The objection can be run as follows.

4.1. $\forall x\forall y\forall z\forall w[($ at $w : x = y \wedge$ at $w$: $y = z) \rightarrow$ at $w : x = z]$ (assumption for reductio).

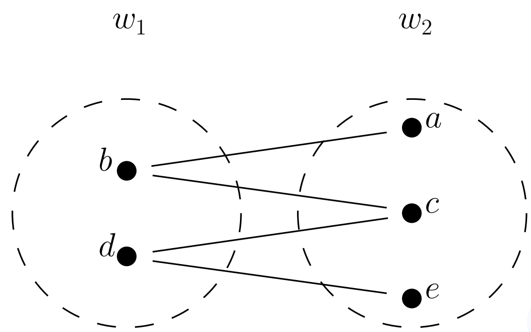


Figure 3: Possible Fission and Fusion of Two Amoebas

4.2. at $w_1 : a = c \land$ at $w_1$: $c = e \land$ at $w_1 : a \neq e$ (from the case).

4.3. (at $w_1$: $a = c \land$ at $w_1$: $c = e$) $\rightarrow$ at $w_1 : a = e$ (instance of 4.1).

4.4. at $w_1 : a = e \land$ at $w_1$: $a \neq e$ (From 4.2 and 4.3, a contradiction).

4.5. $\neg \forall x \forall y \forall z \forall w$[( at $w : x = y \land$ at $w$: $y = z$) $\rightarrow$ at $w : x = z$] (the negation of Transitivity of Identity$_{\text{w}}$).

# 3 Another Transitive Relation

Having reviewed Bader's objections from the Transitivity of Identity, I present how Occasional Identity and Contingent Identity theorists should formulate transitivity principles in general. I do this by, in subsection 3.1, introducing a variation on the case of Amoeba in order to consider the *is to the north of* relation. In subsection 3.2, I show how Occasional Identity theorists ought to reformulate transitivity for this relation. In subsection 3.3, I show how Occasional Identity and Contingent

Identity theorists should reformulate transitivity principles in general, including for the identity relation.

### 3.1 Is To The North Of

Consider the relation *is to the north of.* Like identity, it is a transitive relation. When (i) an object is to the north of a second object and (ii) the second object is to the north of a third, then the first object is to the north of the third. However, unlike identity, it is neither symmetric nor reflexive. If an object is to the north of a second object, then the second object is not to the north of the first object. Also, it is not the case that objects are ever to the north of themselves. So while it is transitive like identity, *is to the north of* is not an equivalence relation like identity.

Recall that in the Amoeba case AMOEBA at $t_1$ divides into POND and SLIDE at $t_2$. Now imagine, as depicted in Figure 4, that between POND and SLIDE at $t_2$ there is a tree, called TREE, such that POND is to the north of TREE, and TREE is to the north of SLIDE. Given that the relation is transitive, we can correctly infer from the fact that POND is to the north of TREE and TREE is to the north of SLIDE, that POND is to the north of SLIDE.

But recall that, because this is a case of amoebic division, the Occasional Identity theorist says that at $t_2$ AMOEBA is SLIDE. So, at $t_2$, TREE is to the north of AMOEBA. Moreover, at $t_2$, AMOEBA is POND. So, at $t_2$, AMOEBA is to the north of TREE. So, by the transitivity of *is to the north of*, at $t_2$ TREE is to the north of TREE.[13] This is contrary to the assumption that *is to the north of* is never reflexive.

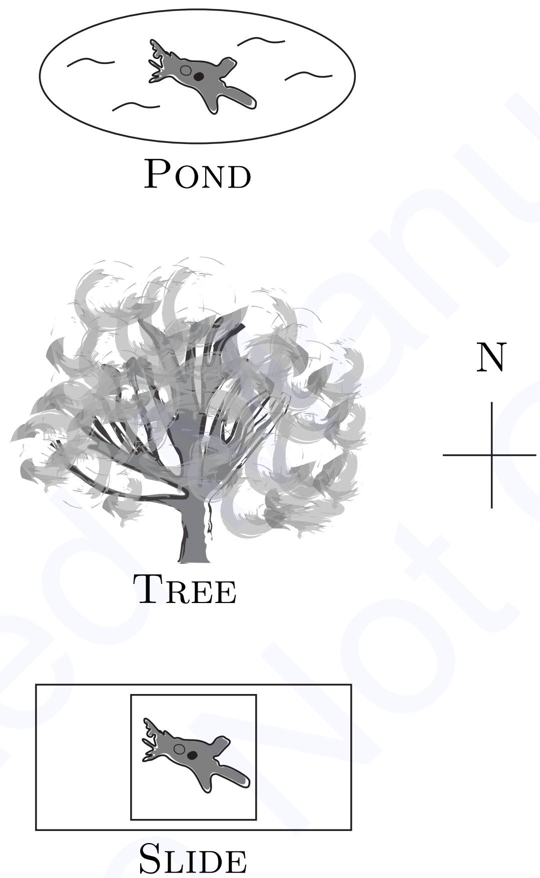


Figure 4: Slide, Tree, and Pond

## 3.2 Reformulating Transitivity

Recall that Gallois says that he accepts the following reformulation of the Transitivity of Identity.

`Transitivity of Identity`$_{\mathtt{t}}$: $\forall x \forall y \forall z \forall t[(\text{ at } t : x = y \wedge \text{at } t\text{: } y = z) \rightarrow \text{at } t : x = z]$

Presumably, he would be prepared to accept the following formulation, where $N$ is the *is to the north of* relation, of a transitivity principle for *is to the north of*.

`Transitivity of N`$_{\mathtt{t}}$: $\forall x \forall y \forall z \forall t[(\text{ at } t : xNy \wedge \text{at } t\text{: } yNz) \rightarrow \text{at } t : xNz]$.

This says that, for all objects and times, when, at a time, (i) one object is to the north of a second and (ii) the second is to the north of a third, then, at that time, the first object is to the north of the third. But this formulation leads us to infer the contradiction presented above. At $t_2$ TREE is to the north of AMOEBA and AMOEBA is to the north of TREE. Then, according to this formulation, at $t_2$, TREE is to the north of itself.

The clue to the correct formulation of transitivity of the *is to the north of* relation for the Occasional Identity theorist comes from a consequence of Gallois' reply to an objection from Saul Kripke (1971). Kripke's argument against Contingent Identity relies on the Necessity of Self-Identity. Gallois imagines and replies to an analogous argument against Occasional Identity that relies on the Eternality of Self-Identity. To respond to this argument, Gallois (1998, Chap. 5) argues that eternal properties should be understood relative to (i) the time at which the object is said to have the eternal property and to (ii) all times.[14]

A consequence of this reply is that Gallois understands the having of temporally indexed properties in terms of the following bi-conditional that I have called Sometime-at-a-time:

`Sometime-at-a-time:` $\Box\forall x\forall t\forall t'[(\text{ at } t\text{: at } t'\text{: } \Phi x) \leftrightarrow \exists y(\text{ at } t\text{: } x = y \wedge \text{at } t'\text{: } \Phi y)]$.[15]

This says that an object has, at a time, the property of having a property at another time just in case, at that time, it is identical to something which, at the other time, has that property.

Gallois's full defense of his interpretation of the predication of temporally indexed properties is too extensive to review here.[16] The following is representative of how he motivates this interpretation. He writes,

> What does it take for it to be true that in 1990 George Bush will be the former President in 2000? ... In 1990 George Bush will be a former President in 2000 if and only if there exists someone who is identical with George Bush in 1990, and who is a former President in 2000. (Gallois, 1998, pp. 83–4)

A consequence of Sometime-at-a-time is that the following bi-conditional holds for the simple predication of a property at one time:

$$\Box\forall x\forall t[(\text{ at } t\text{: at } t\text{: } \Phi x) \leftrightarrow \exists y(\text{ at } t\text{: } x = y \wedge \text{at } t\text{: } \Phi y)]$$

And since the times are the same, this can be simplified so that the following holds:

`Sometime-at-this-time:` $\Box\forall x\forall t[(\text{ at } t\text{: } \Phi x) \leftrightarrow \exists y[\text{ at } t\text{: } (x = y \wedge \Phi y)]]$

The natural analogue of this to two-place relations would then, where $R$ is schematic for two-place relations, be:

$\texttt{Sometime-at-this-time}_{\texttt{R}}\texttt{:}$ $\Box\forall x\forall y\forall t[(\text{ at } t\text{: } xRy) \leftrightarrow \exists x_1\exists y_1[\text{ at } t\text{: } (x = x_1 \wedge y = y_1 \wedge x_1Ry_1)]]$

From this we can generate a first pass at reformulating the transitivity principle for the *is to the north of* relation. This is done by substituting in Transitivity of $\mathrm{N_t}$ instances of the left-hand side of the bi-conditional above with the right-hand side:[17]

$\texttt{Transitivity of N}_{\texttt{te}}\texttt{:}$

$$\forall x\forall y\forall z\forall t[(\exists x_1\exists y_1[\text{ at } t\text{: } (x = x_1 \wedge y = y_1 \wedge x_1Ny_1)]$$
$$\wedge\ \exists y_2\exists z_1[\text{ at } t\text{: } (y = y_2 \wedge z = z_1 \wedge y_2Nz_1)])$$
$$\rightarrow \exists x_2\exists z_2[\text{ at } t\text{: } (x = x_2 \wedge z = z_2 \wedge x_2Nz_2)]]$$

This reformulation says, for all objects $x$, $y$, and $z$, and for all times $t$, when (i) there exists two objects $x_1$ and $y_1$ such that, at time $t$, $x$ is identical to $x_1$, $y$ is identical to $y_1$, and $x_1$ stands in *is to the north of* to $y_1$, and (ii) there exists two objects $y_2$ and $z_1$ such that, at time $t$, $y$ is identical to $y_2$, $z$ is identical to $z_1$, and $y_2$ stands in *is to the north of* to $z_1$, then there exists two objects $x_2$ and $z_2$ such that, at time $t$, $x$ is identical to $x_2$, $z$ is identical to $z_2$, and $x_2$ stands in *is to the north of* to $z_2$.

According to the case, there are two objects such that, at $t_2$, Pond is identical to the first, Tree is identical to the second, and the first object stands in *is to the north of* to the second. Namely, Pond and Tree are two such objects that, at $t_2$, Pond is identical to Pond, Tree is identical to Tree, and Pond stands in *is to* *the north of* to Tree. Also, according to the case, there are two objects such that, at $t_2$, Tree is identical to the first, Slide is identical to the second, and the first object stands in *is to the north of* to the second. Namely, Tree and Slide are two such objects that, at $t_2$, Tree is identical to Tree, Slide is identical to Slide, and Tree stands in *is to the north of* to Slide. According to Transitivity of $\mathrm{N}_{\mathrm{te}}$, we can then infer that there are two objects such that, at $t_2$, Pond is identical to the first, Slide is identical to the second, and the first object stands in *is to the north of* to the second. And according to the case this is true. Namely, Pond and Slide are two such objects that, at $t_2$, Pond is identical to Pond, Slide is identical to Slide, and Pond stands in *is to the north of* to Slide. So far, Transitivity of $\mathrm{N}_{\mathrm{te}}$ delivers a correct inference about the case.

However, according to the case, there are also two objects such that, at $t_2$, Tree is identical to the first, Amoeba is identical to the second, and the first object stands in *is to the north of* to the second. Namely, Tree and Slide are two such objects that, at $t_2$, Tree is identical to Tree, Amoeba is identical to Slide, and Tree stands in *is to the north of* to Slide. And according to the case, there are two objects such that, at $t_2$, Amoeba is identical to the first, Tree is identical to the second, and the first object stands in *is to the north of* to the second. Namely, Pond and Tree are two such objects that, at $t_2$, Amoeba is identical to Pond, Tree is identical to Tree, and Pond stands in *is to the north of* to Tree. From Transitivity of $\mathrm{N}_{\mathrm{te}}$, we should then be able to infer that there are two objects such that, at $t_2$, Tree is identical to the first, Tree is identical to the second, and the first object stands in *is to the north of* to the second. But there are no such objects because TREE is not to the north of itself.

This shows that Transitivity of $N_{te}$ does not capture the transitivity of *is to the north of* because different objects might satisfy the variables $y_1$ and $y_2$ bound by their respective existential quantifiers. If identities are never occasional, this would not be an issue. If objects were never occasionally distinct, there would never be distinct objects to satisfy $y_1$ and $y_2$. However, the example of SLIDE, TREE, and POND shows us that what we might call the 'intermediate' object in an instance of transitivity serves as the intermediate object in virtue of being distinct objects at $t_2$. AMOEBA serves as the intermediate object in the first conjunct of the antecedent of an instance of Transitivity of $N_{te}$ in virtue of being SLIDE at $t_2$. But then it serves as the intermediate object in the second conjunct of the antecedent of Transitivity of $N_{te}$ in virtue, not of being SLIDE, but of being POND at $t_2$.

To properly formulate a transitivity principle for *is to the north of*, we need to bind the pair $y_1$ and $y_2$ to a single existential quantifier. Here is such a formulation:

`Transitivity of` $\mathtt{N_{te*}}$`:`

$$\forall x \forall y \forall z \forall t[\exists x_1 \exists y_1 \exists z_1([\text{ at } t\text{: } (x = x_1 \wedge y = y_1 \wedge x_1 N y_1)]$$
$$\wedge[\text{ at } t\text{: } (y = y_1 \wedge z = z_1 \wedge y_1 N z_1)])$$
$$\rightarrow \exists x_2 \exists z_2[\text{ at } t\text{: } (x = x_2 \wedge z = z_2 \wedge x_2 N z_2)]]$$ [18]

This reformulation says, for all objects $x$, $y$, and $z$, and for all times $t$, when there exists objects $x_1$, $y_1$, and $z_1$ such that when (i), at $t$, $x$ is identical to $x_1$, $y$ is identical to $y_1$, and $x_1$ stands in *is to the north of* to $y_1$, and (ii), at $t$, $y$ is identical to $y_1$, $z$ is identical to $z_1$, and $y_1$ stands in *is to the north of* to $z_1$, then, there exists objects

$x_2$ and $z_2$ such that, at $t$, $x$ is identical to $x_2$, $z$ is identical to $z_2$, and $x_2$ stands in *is to the north of* to $z_2$.

This formulation allows us to make the correct inference about the case (that Pond is to the north of Slide), without making the incorrect one (that Tree is to the north of Tree). This is because, when Pond, Tree, Slide, and $t_2$ satisfy the universal quantifiers, Pond, Tree, and Slide satisfy the first trio of existential quantifiers in Transitivity of $\mathrm{N}_{\mathrm{te}^*}$ such that the antecedent is true and Pond and Slide satisfy the pair of existential quantifiers that make the consequent true. This leads to the correct inference that Pond is to the north of Slide. And because, when Tree, Amoeba, Tree, and $t_2$ satisfy the universal quantifiers, there is no trio of objects to satisfy the existential quantifiers in the antecedent of Transitivity of $\mathrm{N}_{\mathrm{te}^*}$, we do not infer that there is a pair of objects satisfying consequent. And thereby, we are not led to the conclusion that Tree stands in *is to the north of* to itself.

### 3.3 Generalization

Transitivity of $\mathrm{N}_{\mathrm{te}^*}$ suggests the general form that transitivity principles ought to take for Occasional Identity and Contingent Identity theories. Here are the schemas for the temporal and modal versions of transitivity where $R$ is the relation in question:

`Transitivity of R`$_{\mathtt{te*}}$:

$$\forall x \forall y \forall z \forall t[\exists x_1 \exists y_1 \exists z_1([\text{ at } t\text{: } (x = x_1 \wedge y = y_1 \wedge x_1 R y_1)]$$
$$\wedge[\text{ at } t\text{: } (y = y_1 \wedge z = z_1 \wedge y_1 R z_1)])$$
$$\rightarrow \exists x_2 \exists z_2[\text{ at } t\text{: } (x = x_2 \wedge z = z_2 \wedge x_2 R z_2)]]$$

`Transitivity of R`$_{\texttt{we*}}$`:`

$\forall x \forall y \forall z \forall w[\exists x_1 \exists y_1 \exists z_1([\text{ at } w\text{: } (x = x_1 \wedge y = y_1 \wedge x_1 R y_1)]$

$\wedge[\text{ at } w\text{: } (y = y_1 \wedge z = z_1 \wedge y_1 R z_1)])$

$\rightarrow \exists x_2 \exists z_2[\text{ at } w\text{: } (x = x_2 \wedge z = z_2 \wedge x_2 R z_2)]]$

Transitivity of $\mathrm{R}_{\mathrm{te}*}$ says that, for all objects $x$, $y$, and $z$, and for all times $t$, when there exists objects $x_1$, $y_1$, and $z_1$ such that when (i), at $t$, $x$ is identical to $x_1$, $y$ is identical to $y_1$, and $x_1$ stands in $R$ to $y_1$, and (ii), at $t$, $y$ is identical to $y_1$, $z$ is identical to $z_1$, and $y_1$ stands in $R$ to $z_1$, then, there exists objects $x_2$ and $z_2$ such that, at $t$, $x$ is identical to $x_2$, $z$ is identical to $z_2$, and $x_2$ stands in $R$ to $z_2$.

Similarly for Transitivity of $\mathrm{R}_{\mathrm{we}*}$, except where $t$ is substituted with $w$.

With the general forms of transitivity, we can specify the temporal and modal reformulations of Transitivity of Identity as follows.

`Transitivity of Identity`$_{\texttt{te*}}$`:`

$\forall x \forall y \forall z \forall t[\exists x_1 \exists y_1 \exists z_1([\text{ at } t\text{: } (x = x_1 \wedge y = y_1 \wedge x_1 = y_1)]$

$\wedge[\text{ at } t\text{: } (y = y_1 \wedge z = z_1 \wedge y_1 = z_1)])$

$\rightarrow \exists x_2 \exists z_2[\text{ at } t\text{: } (x = x_2 \wedge z = z_2 \wedge x_2 = z_2)]]$

`Transitivity of Identity`$_{\texttt{we*}}$`:`

$\forall x \forall y \forall z \forall w[\exists x_1 \exists y_1 \exists z_1([\text{ at } w\text{: } (x = x_1 \wedge y = y_1 \wedge x_1 = y_1)]$

$\wedge[\text{ at } w\text{: } (y = y_1 \wedge z = z_1 \wedge y_1 = z_1)])$

$\rightarrow \exists x_2 \exists z_2[\text{ at } w\text{: } (x = x_2 \wedge z = z_2 \wedge x_2 = z_2)]]$

Transitivity of $\mathrm{Identity}_{\mathrm{te}*}$ says that, for all objects $x$, $y$, and $z$, and for all times $t$, when there exists objects $x_1$, $y_1$, and $z_1$ such that when (i), at $t$, $x$ is identical to

$x_1$, $y$ is identical to $y_1$, and $x_1$ is identical to $y_1$, and (ii), at $t$, $y$ is identical to $y_1$, $z$ is identical to $z_1$, and $y_1$ is identical to $z_1$, then, there exists objects $x_2$ and $z_2$ such that, at $t$, $x$ is identical to $x_2$, $z$ is identical to $z_2$, and $x_2$ is identical to $z_2$.

Similarly for Transitivity of Identity$_{\mathrm{we}^*}$, except where $t$ is substituted with $w$.

The next section shows how these reformulations of Transitivity of Identity provide a reply to Bader's objections to Occasional Identity and Contingent Identity.

# 4 Replying to Bader

Recall Bader's objections to Occasional Identity and Contingent Identity. Here they are now formulated with Transitivity of Identity$_{\mathrm{te}^*}$ and Transitivity of Identity$_{\mathrm{we}^*}$, repsectively:

3.1* $\forall x \forall y \forall z \forall t[\exists x_1 \exists y_1 \exists z_1([\text{ at } t\text{: } (x = x_1 \wedge y = y_1 \wedge x_1 = y_1)]$
$\wedge[\text{ at } t\text{: } (y = y_1 \wedge z = z_1 \wedge y_1 = z_1)])$
$\rightarrow \exists x_2 \exists z_2[\text{ at } t\text{: } (x = x_2 \wedge z = z_2 \wedge x_2 = z_2)]]$ (assumption for reductio).

3.2* at $t_1$:$(a = b \wedge c = b \wedge a = c) \wedge$ at $t_1$:$(c = d \wedge e = d \wedge c = e) \wedge$
$\neg\exists x \exists z[\text{at } t_1\text{:}(a = x \wedge e = z \wedge x = z)]$ (from the case).

3.3* $\exists x_1 \exists y_1 \exists z_1([\text{ at } t_1\text{: } (a = x_1 \wedge c = y_1 \wedge x_1 = y_1)]$
$\wedge[\text{ at } t_1\text{: } (c = y_1 \wedge e = z_1 \wedge y_1 = z_1)])$
$\rightarrow \exists x_2 \exists z_2[\text{ at } t\text{: } (a = x_2 \wedge e = z_2 \wedge x_2 = z_2)]$ (an instance of 3.1*).

3.4* $\exists x \exists z[\text{at } t_1\text{:}(a = x \wedge e = z \wedge x = z)] \wedge \neg\exists x \exists z[\text{at } t_1\text{:}(a = x \wedge e = z \wedge x = z)]$
(from 3.2* and 3.3*, a contradiction).

3.5* $\neg\forall x\forall y\forall z\forall t[\exists x_1\exists y_1\exists z_1([\text{ at } t\text{: } (x = x_1 \wedge y = y_1 \wedge x_1 = y_1)]$

$\wedge[\text{ at } t\text{: } (y = y_1 \wedge z = z_1 \wedge y_1 = z_1)])$

$\rightarrow \exists x_2\exists z_2[\text{ at } t\text{: } (x = x_2 \wedge z = z_2 \wedge x_2 = z_2)]]$ (the negation of Transitivity of Identity$_{\text{te*}}$).

and

4.1* $\forall x\forall y\forall z\forall w[\exists x_1\exists y_1\exists z_1([\text{ at } w\text{: } (x = x_1 \wedge y = y_1 \wedge x_1 = y_1)]$

$\wedge[\text{ at } w\text{: } (y = y_1 \wedge z = z_1 \wedge y_1 = z_1)])$

$\rightarrow \exists x_2\exists z_2[\text{ at } w\text{: } (x = x_2 \wedge z = z_2 \wedge x_2 = z_2)]]$ (assumption for reductio).

4.2* at $w_1$:$(a = b \wedge c = b \wedge a = c) \wedge$ at $w_1$:$(c = d \wedge e = d \wedge c = e) \wedge$

$\neg\exists x\exists z$[at $w_1$:$(a = x \wedge e = z \wedge x = z)$] (from the case).

4.3* $\exists x_1\exists y_1\exists z_1([\text{ at } w_1\text{: } (a = x_1 \wedge c = y_1 \wedge x_1 = y_1)]$

$\wedge[\text{ at } w_1\text{: } (c = y_1 \wedge e = z_1 \wedge y_1 = z_1)])$

$\rightarrow \exists x_2\exists z_2[\text{ at } w\text{: } (a = x_2 \wedge e = z_2 \wedge x_2 = z_2)]$ (an instance of 4.1*).

4.4* $\exists x\exists z$[at $w_1$:$(a = x \wedge e = z \wedge x = z)$] $\wedge\, \neg\exists x\exists z$[at $w_1$:$(a = x \wedge e = z \wedge x = z)$]

(from 4.2* and 4.3*, a contradiction).

4.5* $\neg\forall x\forall y\forall z\forall w[\exists x_1\exists y_1\exists z_1([\text{ at } w\text{: } (x = x_1 \wedge y = y_1 \wedge x_1 = y_1)]$

$\wedge[\text{ at } w\text{: } (y = y_1 \wedge z = z_1 \wedge y_1 = z_1)])$

$\rightarrow \exists x_2\exists z_2[\text{ at } w\text{: } (x = x_2 \wedge z = z_2 \wedge x_2 = z_2)]]$ (the negation of Transitivity of Identity$_{\text{we*}}$).

The arguments with the reformulated versions of the Transitivity of Identity are not valid. In particular, the inference from lines 2 and 3 to line 4 in each is not valid.

This is because the first two conjuncts in lines 2 do not make the antecedent in line 3 true. This is because $b$ and $d$ are not the same at $t_1$ or $w_1$, and thereby cannot thereby satisfy the variable $y_1$ bound by the second existential quantifier in line 3. Here are lines 3.2* and 4.2* repeated with the distinct objects $b$ and $b$ bolded to show that they cannot satisfy the variable $y_1$:

3.2* at $t_1$:$(a = b \wedge c = \mathbf{b} \wedge a = c) \wedge$ at $t_1$:$(c = \mathbf{d} \wedge e = d \wedge c = e) \wedge$
$\neg\exists x \exists z$[at $t_1$:$(a = x \wedge e = z \wedge x = z)$]

4.2* at $w_1$:$(a = b \wedge c = \mathbf{b} \wedge a = c) \wedge$ at $w_1$:$(c = \mathbf{d} \wedge e = d \wedge c = e) \wedge$
$\neg\exists x \exists z$[at $w_1$:$(a = x \wedge e = z \wedge x = z)$]

Now that the inference from line 2 to 3 is invalid, the inference to line 4 is not justified. That is, *modus ponens* can no longer be used to infer the first conjunct in 4 from lines 2 and 3. And without the contradiction in line 4 the *reductio* does not go through.

# 5 Conclusion

The reformulations Transitivity of Identity$_{\text{te}*}$ and Transitivity of Identity$_{\text{we}*}$ expose how Bader's objections in subsection 2.4 worked. As explained in subsection 3.2, they trade one identity in the first conjunct of the antecedents in the transitivity principles for another identity in the second conjunct. These formulations force the identities to be the same. Bader's argument attempted to show that according to Occasional Identity and Contingent Identity identity was not transitive, and that by rejecting one of the Metaphysical Principles of Identity they have incurred the further cost, one that many would take to be an unacceptable cost, of rejecting one of the Logical Principles of Identity.

By reformulating the transitivity principles as I have suggested, Occasional Identity and Contingent Identity can maintain that identity is transitive on their views. Moreover, the generalization shown in subsection 3.3 provides the schema for reformulating principles for any transitive relations under these theories. Reformulating Principles of Identity is consistent with the strategy that Gallois has already employed to respond to objections to his view. Further, I have proposed formulations in light of his own views about temporally and modally indexed properties. Bader is right to point out that the formulations of transitivity that he considers are inadequate given the possibility of simultaneous fissions and fusion. However, he has failed to show that Occasional Identity and Contingent Identity theorists cannot provide adequate reformulations of the Transitivity of Identity. Arguably, there are many theoretical costs for adopting Occasional Identity and Contingent Identity, but rejecting that identity is transitive is not among them.

# Notes

1. See Gallois 1998, pp. 69–70 for an argument that Occasional Identity implies Contingent Identity.

2. Names of objects are typeset in small-caps throughout. Principles and views are monospaced when first mentioned or defined.

3. A relation is said to be reflexive when every object stands in the relation to itself. More precisely

and with respect to the identity relation:

`Reflexivity of Identity:` $\forall x(x = x)$

A relation is said to be symmetric when, for two objects, the first stands in the relation to the second just in case the second stands in it to the first. More precisely and with respect to identity:

`Symmetry of Identity` $\forall x \forall y(x = y \rightarrow y = x)$

4. See, for example, Griffin 1977 for an argument that identity is relative, rather than absolute. See, for example, Parsons 2000 for an argument that identity is indeterminate, rather than absolute. See, for example, Cotnoir & Baxter 2014; Wallace 2011a,b for discussions of the view that identity is composition, and as such is a many-one relation.

5. Here I do not say whether Leibniz's Law is one of the Logical Principles of Identity or Metaphysical Principles of Identity. I think there are good reasons for either categorization. In [redacted for review] I claimed that, when it comes to theory choice, it functions like the Logical Principles of Identity.

6. By "a case of Occasional Identity/Contingent Identity" I mean a case that, if it were true, would be sufficient for the truth of Occasional Identity or Contingent Identity, respectively.

7. The opponent of Contingent Identity would justify 1.3 and 1.4 by Reflexivity of Identity. Gallois (1998, pp. 76–7) would agree that 1.3 and 1.4 are true by the particulars of the case, but not in virtue of Reflexivity of Identity. While not discussed in Gallois 1998, an Occasional Identity theorist would, for reasons similar to relativizing Transitivity of Identity, only accept a reformulation of Reflexivity of Identity that fixes the principle to the same time. See Gallois 1998, pp. 91–2 for a discussion of reflexivity.

8. The argument has been adapted. The original uses descriptions instead of names to illustrate a different point.

9. Where @ is the actual world.

10. See above for a discussion of the justification for 2.3 and 2.4.

11. See Gilmore 2009 for an example of relativizing transitivity for his view that the parthood relation is four-place.

12. See van Inwagen 1981 for an argument against the assumption underlying this view.

13. It is also the case that by transitivity, Amoeba is to the north of Amoeba. This is because Amoeba is Pond which is to the north of Tree which is to the north of Slide which is identical to Amoeba.

14. The particular formulation of this is given on p. 129. And for structurally similar reasons, he argues that necessary properties should be understood relative to (i) the world at which the object is said to have the necessary property and to (ii) all worlds (Gallois, 1998, Chap. 6).

15. When presented (on p. 84), this is considered along with a variation that replaces the existential quantifier that binds the $y$ variable in the right-hand side of the biconditional with a universal quantifier. On this understanding of temporally indexed properties, to have such a property is for everything that an object is identical to at that time to have the property at another time. Arguably, while objects might have temporally indexed properties in this sense, these construals cannot capture some temporally indexed properties that Occasional Identity theorists might want to capture. Gallois officially accepts that there might be some properties that are best captured under this formulation. However, in the case of eternal properties, he argues that the formulation with an existential quantifier best captures what it means for an object to have eternal properties. Further, he suggests that his interpretations of eternal properties and temporally indexed properties naturally come together. If this is so, then there is reason to accept the existential formulation rather than the universally quantified one.

16. Such defenses are given in Chaps. 3 and 5.

17. I have argued in [redacted for review] that Gallois could harmonize the response I propose here with

his preferred view of instantiation (Gallois, 1998, p. 38). Thank you to an anonymous reviewer for this journal for pointing out that the particular proposal presented there could be separated from the response to Bader's objection.

18. It might be suggested that we should also bind $x_1$ and $x_2$ to the same quantifier, and further, bind $z_1$ and $z_2$ to the same quantifier. Although I do not think a counterexample demonstrating the inadequacy of this current formulation is forthcoming, I do not see why such a reformulation that binds those variables would be objectionable. Such a formulation was in fact suggested by an anonymous reviewer for this journal.

# References


Bader, R. M. (2012). The non-transitivity of the contingent and occasional identity relations. *Philosophical Studies*, *157*, 141–52.

Cotnoir, A. J. & Baxter, D. L. M. (Eds.) (2014). *Composition as Identity.* Oxford: Oxford University Press.

Frege, G. (1892). Über sinn und bedeutung. *Zeitschrift fur Philosophie und philosophische Kritik*, *100*, 25–50. Translated as "On Sinn and Bedeutung" by Max Black in *Translations from the Philosophical Writings of Gottlob Frege*, Peter Geach and Max Black (Eds.). Oxford: Blackwell, 1952. Reprinted in Micheal Beaney (Ed.), *The Frege Reader*, pp. 151–71. Oxford: Blackwell (1997).

Gallois, A. (1998). *Occasions of Identity: A Study in the Metaphysics of Persistence, Change, and Sameness.* Oxford: Clarendon Press.

Gibbard, A. (1975). Contingent identity. *Journal of Philosophical Logic*, *4*, 187–222.

Gilmore, C. (2009). Why parthood might be a four-place relation, and how it behaves if it is. In L. Honnefelder, E. Runggaldier, & B. Schick (Eds.), *Unity and Time in Metaphysics*, (83–133). Berlin: Walter de Gruyter.

Griffin, N. (1977). *Relative Identity*. Oxford: Clarendon Press.

Hawthorne, J. (2003). Identity. In M. J. Loux & D. W. Zimmerman (Eds.), *The Oxford Handbook of Metaphysics*, (99–130). Oxford: Oxford University Press.

Kripke, S. A. (1971). Identity and necessity. In M. K. Munitz (Ed.), *Identity and Individuation*, (135–64). New York: New York University Press.

Parsons, T. (2000). *Indeterminate Identity: Metaphysics and Semantics*. Oxford: Clarendon Press.

van Inwagen, P. (1981). The doctrine of arbitrary undetached parts. *Pacific Philosophical Quarterly*, *62*, 123–37.

Wallace, M. (2011a). Composition as identity: Part 1. *Philosophy Compass*, *6*, 804–16.

Wallace, M. (2011b). Composition as identity: Part 2. *Philosophy Compass*, *6*, 817–27.